

Express each of the following decimals in the form $\frac{p}{q}$:

- (i) $0.\overline{4}$ (ii) $0.\overline{37}$
 (iii) $0.\overline{54}$ (iv) $0.\overline{621}$
 (v) $125.\overline{3}$ (vi) $4.\overline{7}$
 (vii) $0.4\overline{7}$

Examine, whether the following numbers are rational or irrational:

- (i) $\sqrt{7}$ (ii) $\sqrt{4}$
 (iii) $2 + \sqrt{3}$ (iv) $\sqrt{3} + \sqrt{2}$
 (v) $\sqrt{3} + \sqrt{5}$ (vi) $(\sqrt{2} - 2)^2$
 (vii) $(2 - \sqrt{2})(2 + \sqrt{2})$ (viii) $(\sqrt{2} + \sqrt{3})^2$
 (ix) $\sqrt{5} - 2$ (x) $\sqrt{23}$
 (xi) $\sqrt{225}$ [NCERT] (xii) 0.3796
 (xiii) $7.478478.....$
 (xiv) $1.101001000100001.....$

In the following equation, find which variables x, y, z etc. represent rational or irrational numbers:

- (i) $x^2 = 5$ (ii) $y^2 = 9$
 (iii) $z^2 = 0.04$ (iv) $u^2 = \frac{17}{4}$
 (v) $v^2 = 3$ (vi) $w^2 = 27$
 (vii) $t^2 = 0.4$

Simplify the following:

- (i) $3(a^4b^3)^{10} \times 5(a^2b^2)^3$ (ii) $(2x^2y^3)^3$ (iii) $\frac{(4 \times 10^7)(6 \times 10^{-5})}{8 \times 10^4}$
 (iv) $\frac{4ab^2(-5ab^3)}{10a^2b^2}$ (v) $\left(\frac{x^2y^2}{a^2b^3}\right)^n$ (vi) $\frac{(a^{3n-9})^6}{a^{2n-4}}$

If $a = 3$ and $b = -2$, find the values of:

- (i) $a^a + b^b$
- (ii) $a^b + b^a$
- (iii) $(a+b)^{ab}$

Prove that:

$$(i) \left(\frac{x^a}{x^b} \right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c} \right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a} \right)^{c^2+ca+a^2} = 1$$

$$(ii) \left(\frac{x^a}{x^b} \right)^c \times \left(\frac{x^b}{x^c} \right)^a \times \left(\frac{x^c}{x^a} \right)^b = 1$$

$$(iii) \left(\frac{x^a}{x^b} \right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^c} \right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^a} \right)^{c^2-ca+a^2} = 1$$

Prove that

$$(i) \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$$

$$(ii) \frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}} = 1$$

Prove that

$$(i) \frac{a+b+c}{a^{-1}b^{-1}+b^{-1}c^{-1}+c^{-1}a^{-1}} = abc \quad (ii) (a^{-1}+b^{-1})^{-1} = \frac{ab}{a+b}$$

If $abc = 1$, show that $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}} = 1$

Simplify the following:

$$(i) \frac{3n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$$

$$(ii) \frac{5 \times 25^{n+1} - 25 \times 5^{2n}}{5 \times 5^{2n+3} - (25)^{n+1}}$$

$$(iii) \frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 2^2 \times 5^n}$$

$$(iv) \frac{6(8)^{n+1} + 16(2)^{3n-2}}{10(2)^{3n+1} - 7(8)^n}$$

Solve the following equations for x:

$$(i) 7^{2x+3} = 1$$

$$(ii) 2^{x+1} = 4^{x-3}$$

$$(iii) 2^{5x+3} = 8^{x+3}$$

$$(iv) 4^{2x} = \frac{1}{32}$$

$$(v) 4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$

$$(vi) 2^{3x-7} = 256$$

If $49392 = a^4 b^2 c^3$, find the values of a, b and c, where a, b and c are different positive primes.

Assuming that x, y, z are positive real numbers, simplify each of the following:

$$(i) \left(\sqrt{x^{-3}}\right)^5$$

$$(ii) \sqrt{x^3 y^{-2}}$$

$$(iii) (x^{-2/3} y^{-1/2})^2$$

$$(iv) (\sqrt{x})^{-2/3} \sqrt{y^4} \div \sqrt{xy^{-1/2}}$$

$$(v) \sqrt[5]{243x^{10}y^5z^{10}}$$

$$(vi) \left(\frac{x^{-4}}{y^{-10}}\right)^{5/4}$$

$$(vii) \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^5 \left(\frac{6}{7}\right)^2$$

Simplify:

$$(i) (16^{-1/5})^{5/2} \quad (ii) \sqrt[5]{(32)^{-3}}$$

$$(iii) \sqrt[3]{(343)^{-2}} \quad (iv) (0.001)^{1/3}$$

$$(v) \frac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}}$$

$$(vi) \left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13}$$

$$(vii) \left(\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right)^{\frac{-5}{2}}$$

Prove that:

$$(i) \sqrt{3 \times 5^{-3}} \div \sqrt[3]{3^{-1}} \sqrt{5} \times \sqrt[6]{3 \times 5^6} = \frac{3}{5}$$

$$(ii) 9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{\frac{-1}{2}} = 15$$

$$(iii) \left(\frac{1}{4}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{\frac{-1}{2}} = \frac{16}{3}$$

$$(iv) \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{\frac{-1}{5}} \times 5^{\frac{3}{5}}} \div \frac{3^{\frac{4}{3}} \times 5^{\frac{-7}{5}}}{4^{\frac{-3}{5}} \times 6} = 10$$

$$(v) \sqrt{\frac{1}{4}} + (0.01)^{-1/2} - (27)^{2/3} = \frac{3}{2}$$

$$(vi) \frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{3}{2}$$

$$(vii) \left(\frac{64}{125}\right)^{\frac{-2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0 = \frac{61}{16}$$

$$(viii) \frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{1/25} \times (15)^{-4/3} \times 3^{1/3}} = 28\sqrt{2}$$

$$(ix) \frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1} \left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}} = -\frac{3}{2}$$

Show that:

$$(i) \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$$

$$(ii) \left[\left\{ \frac{x^{a(a-b)}}{x^{a(a+b)}} \right\} \div \left\{ \frac{x^{b(b-a)}}{x^{b(b+a)}} \right\} \right]^{a+b} = 1$$

$$(iii) \left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}} = 1$$

$$(iv) \left(\frac{x^{a^2+b^2}}{x^{ab}} \right)^{a+b} \left(\frac{x^{b^2+c^2}}{x^{bc}} \right)^{b+c} \left(\frac{x^{c^2+a^2}}{x^{ac}} \right)^{a+c} =$$

$$x^{2(a^3+b^3+c^3)}$$

$$(v) (x^{a-b})^{a+b} (x^{b-c})^{b+c} (x^{c-a})^{c+a} = 1$$

$$(vi) \left\{ \left(x^{a-a^{-1}} \right)^{\frac{1}{a-1}} \right\}^{\frac{a}{a+1}} = x$$

$$(vii) \left(\frac{a^{x+1}}{a^{y+1}} \right)^{x+y} \left(\frac{a^{y+2}}{a^{z+2}} \right)^{y+z} \left(\frac{a^{z+3}}{a^{x+3}} \right)^{z+x} = 1$$

$$(viii) \left(\frac{3^a}{3^b} \right)^{a+b} \left(\frac{3^b}{3^c} \right)^{b+c} \left(\frac{3^c}{3^a} \right)^{c+a} = 1$$

If $2^x = 3^y = 6^z$, show that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$.

If $2^x = 3^y = 12^z$, show that $\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$.

Find the values of x in each of the following:

$$(i) 2^{5x} \div 2^x = \sqrt[5]{2^{20}} \quad (ii) (2^3)^4 = (2^2)^x$$

$$(iii) \left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27} \quad (iv) 5^{x-2} \times 3^{2x-3} = 135$$

$$(v) 2^{x-7} \times 5^{x-4} = 1250 \quad (vi) (\sqrt[3]{4})^{2x+\frac{1}{2}} = \frac{1}{32}$$

$$(vii) 5^{2x+3} = 1 \quad (viii) (13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$

$$(ix) \left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$$

Solve the following equations:

$$(i) 3^{x+1} = 27 \times 3^4$$

$$(ii) 4^{2x} = (\sqrt[3]{16})^{-6/y} = (\sqrt{8})^2$$

$$(iii) 3^{x-1} \times 5^{2y-3} = 225$$

$$(iv) 8^{x+1} = 16^{y+2} \text{ and, } \left(\frac{1}{2}\right)^{3+x} = \left(\frac{1}{4}\right)^{3y}$$

$$(v) 4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$

$$(vi) \sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-2x}, \text{ where } a, b \text{ are distinct positive primes.}$$

Simplify:

$$(i) \left(\frac{x^{a+b}}{x^c} \right)^{a-b} \left(\frac{x^{b+c}}{x^a} \right)^{b-c} \left(\frac{x^{c+a}}{x^b} \right)^{c-a}$$

$$(ii) \sqrt[lm]{\frac{x^l}{x^m}} \times \sqrt[mn]{\frac{x^m}{x^n}} \times \sqrt[nl]{\frac{x^n}{x^l}}$$

- (i) If $a = x^{m+n}y^l$, $b = x^{n+l}y^m$ and $c = x^{l+m}y^n$, prove that $a^{m-n} b^{n-l} c^{l-m} = 1$.
- (ii) If $x = a^{m+n}$, $y = a^{n+l}$ and $z = a^{l+m}$, prove that $x^m y^n z^l = x^n y^l z^m$.

$$\text{Simplify : } \left[\left\{ (625)^{\frac{-1}{2}} \right\}^{\frac{-1}{4}} \right]^2.$$

Simplify each of the following:

$$(i) \sqrt[3]{4} \times \sqrt[3]{16} \quad (ii) \frac{\sqrt[4]{1250}}{\sqrt[4]{2}}$$

Simplify the following expressions:

- (i) $(4 + \sqrt{7})(3 + \sqrt{2})$
 (ii) $(3 + \sqrt{3})(5 - \sqrt{2})$
 (iii) $(\sqrt{5} - 2)(\sqrt{3} - \sqrt{5})$

Simplify the following expressions:

- (i) $(11 + \sqrt{11})(11 - \sqrt{11})$
 (ii) $(5 + \sqrt{7})(5 - \sqrt{7})$
 (iii) $(\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2})$

Simplify the following expressions:

(i) $(\sqrt{3} + \sqrt{7})^2$

(ii) $(\sqrt{5} - \sqrt{3})^2$

(iii) $(2\sqrt{5} + 3\sqrt{2})^2$

Rationalise the denominators of each of the following (i – vii):

(i) $\frac{3}{\sqrt{5}}$

(ii) $\frac{3}{2\sqrt{5}}$

(iii) $\frac{1}{\sqrt{12}}$

(iv) $\frac{\sqrt{2}}{\sqrt{5}}$

(v) $\frac{\sqrt{3} + 1}{\sqrt{2}}$

(vi) $\frac{\sqrt{2} + \sqrt{5}}{\sqrt{3}}$

(vii) $\frac{3\sqrt{2}}{\sqrt{5}}$

Find the value to three places of decimals of each of the following. It is given that

$$\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236$$

$$\text{and } \sqrt{10} = 3.162.$$

(i) $\frac{2}{\sqrt{3}}$

(ii) $\frac{3}{\sqrt{10}}$

(iii) $\frac{\sqrt{5} + 1}{\sqrt{2}}$

(iv) $\frac{\sqrt{10} + \sqrt{15}}{\sqrt{2}}$

(v) $\frac{2 + \sqrt{3}}{3}$

(vi) $\frac{\sqrt{2} - 1}{\sqrt{5}}$

Express each one of the following with rational denominator:

$$(i) \frac{1}{3+\sqrt{2}}$$

$$(ii) \frac{1}{\sqrt{6}-\sqrt{5}}$$

$$(iii) \frac{16}{\sqrt{41}-5}$$

$$(iv) \frac{30}{5\sqrt{3}-3\sqrt{5}}$$

$$(v) \frac{1}{2\sqrt{5}-\sqrt{3}}$$

$$(vi) \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$$

$$(vii) \frac{6-4\sqrt{2}}{6+4\sqrt{2}}$$

$$(viii) \frac{3\sqrt{2}+1}{2\sqrt{5}-3}$$

$$(ix) \frac{b^2}{\sqrt{a^2+b^2}+a}$$

Rationalise the denominator and simplify:

$$(i) \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$(ii) \frac{5+2\sqrt{3}}{7+4\sqrt{3}}$$

$$(iii) \frac{1+\sqrt{2}}{3-2\sqrt{2}}$$

$$(iv) \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$$

$$(v) \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

$$(vi) \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$$

Simplify:

$$(i) \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$(iii) \frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$$

$$(v) \frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$$

In each of the following determine rational numbers a and b:

$$(i) \frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$$

$$(ii) \frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b}$$

$$(iii) \frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$$

$$(iv) \frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$$

$$(v) \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$$

$$(vi) \frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$$

Find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$, it being given
that $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$.

Find the values of each of the following correct to three places of decimals, it being given that $\sqrt{2} = 1.4142$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.2360$, $\sqrt{6} = 2.4495$ and $\sqrt{10} = 3.162$.

$$(i) \frac{3-\sqrt{5}}{3+2\sqrt{5}} \quad (ii) \frac{1+\sqrt{2}}{3-2\sqrt{2}}$$

Simplify:

$$(i) \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$$

$$(ii) \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

If $x = 2 + \sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$.

If $x = 3 + \sqrt{8}$, find the value of $x^2 + \frac{1}{x^2}$.